Tutorial 1: Mechanics of Particles:

- Laws of motion; conservation of energy and momentum,
- applications to rotating frames, centripetal and Coriolis accelerations;
- Conservation of angular momentum,

Q1.

Suppose that an S′-frame is rotating with respect to a fixed frame having the same origin. Assume that the angular velocity $\vec{\omega}$ of the S′-frame is given by

$$\vec{\omega} = 2t\,\hat{i} - t^2\,\hat{j} + (2t + 4)\,\hat{k}$$

where t is time and the position vector $\vec{r}$ of a typical particle at time t as assumed in S′-frame is given by

$$\vec{r} = (t^2 + 1)\,\hat{i} - 6\,t\,\hat{j} + 4\,t^3\,\hat{k}.$$

Calculate the Coriolis acceleration at t = 1 second. *10*

2013

Q2.

(a) If the forces acting on a particle are conservative, show that the total energy of the particle which is the sum of the kinetic and potential energies is conserved. *20*

(b) Prove that as a result of an elastic collision of two particles under non-relativistic regime with equal masses, the scattering angle will be 90°. Illustrate your answer with a vector diagram. *5*

(c) Calculate the horizontal component of the Coriolis force acting on a body of mass 0·1 kg moving northward with a horizontal velocity of 100 ms^{-1} at 30° N latitude on the Earth. *15*

2013

Q3.

Write down precisely the conservation theorems for energy, linear momentum and angular momentum of a particle with their mathematical forms. *10*

2015

Q4:

How does one obtain the angular velocity of the Earth about the North Pole with respect to a fixed star as $7{\cdot}292 \times 10^{-5}$ sec^{-1} ? Explain your method of calculating the above value. *10*

2015

Q5:

Derive the expression for Coriolis force and show that this force is perpendicular to the velocity and to the axis of rotation. What is the nature of this force? 10

2016

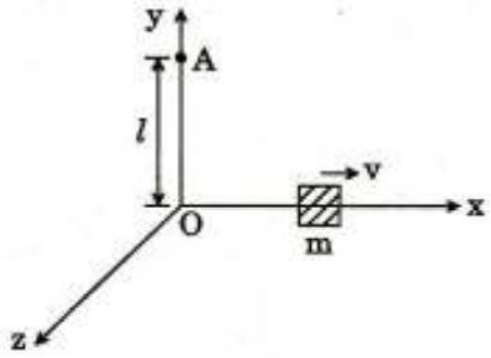
Q6:

A ball moving with a speed of 9 m/s strikes an identical stationary ball such that after the collision the direction of each ball makes an angle 30° with the original line of motion. Find the speed of the balls after the collision. Is the kinetic energy conserved in this collision? 15

2017

Q7:

A block of mass m having negligible dimension is sliding freely in x-direction with velocity $\vec{v} = v\hat{i}$ as shown in the diagram. What is its angular momentum $\vec{L}_O$ about origin O and its angular momentum $\vec{L}_A$ about the point A on y-axis ? *10*



2017

Q8:

A rocket starts vertically upward with speed v_0. Then define its speed v at a height h in terms of v_0, h, R (radius of Earth) and g (acceleration due to gravity on Earth's surface).
Also calculate the maximum height attained by a rocket fired with a speed of 90% of the escape velocity. *10*

2020

Q9:

Consider two frames of reference S and S' having a common origin O. The frame S' is rotating with respect to the fixed frame S with a uniform $\vec{\omega} = 3a_x$ rad s^{-1}. A projectile of unit mass at position vector $\vec{r} = 7a_x + 4a_y$ m is moving with $\vec{v} = 14a_y$ m s^{-1}. Calculate in the rotating frame S' the following forces on the projectile :

(i) Euler's force

(ii) Coriolis force

(iii) Centrifugal force 15

2022

Q10:

A force $\vec{F}$ is given by $\vec{F} = x^2y\hat{x} + zy^2\hat{y} + xz^2\hat{z}$. Determine whether or not the force is conservative. *10*

2023

Q11.

Show that the kinetic energy of a system of n particles is given by

$$T = \frac{1}{2}MV_{cm}^2 + \frac{1}{2}\sum_{i=1}^{n} m_i V_i'^2$$

where M is the total mass, V_{cm} is the velocity of the centre of mass, V_i' is the velocity of the particles about the centre of mass and m_i is the mass of the ith particle. 10

2024

Q12.

A particle of mass m kg having an initial velocity V_0 is subjected to a retarding force proportional to its instantaneous velocity. Obtain the expression for the velocity and position of the particle as a function of time. 10

2024