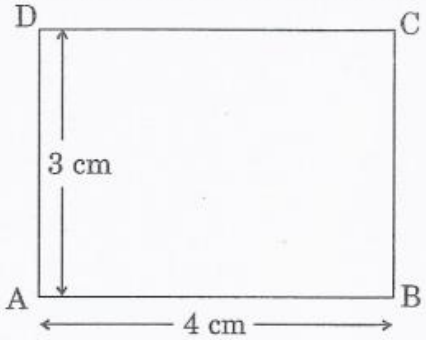
Tutorial 1: Electrostatics

- Laplace and Poisson equations in electrostatics and their applications;
- Energy of a system of charges, multipole expansion of scalar potential;
- Method of images and its applications;
- Potential and field due to a dipole, force and torque on a dipole in an external field;
- Dielectrics, polarization; Solutions to boundary-value problems-conducting and dielectric spheres in a uniform electric field;

Q1:

(c) Using the fundamental concepts of electromagnetism, determine the electric field of an electric dipole $\vec{p}$ at a distance $\vec{r}$ and its energy in an electric field $\vec{E}$. *15*

(d) ABCD is a rectangle in which charges of $+10^{-11}$ C, -2×10^{-11} C and 10^{-11} C are placed at corners B, C and D, respectively.



Calculate the potential at the corner A and the work done in carrying a charge of 2 coulombs to A. *10*

2013

Q2:

Under one-dimensional configuration, the charge density is given by $\rho(x) = \frac{\rho_0 x}{5}$; where ρ_0 is a constant charge density. If the electric field $|\vec{E}| = 0$ at $x = 0$ and potential $V = 0$ at $x = 5$, determine V and $|\vec{E}|$. *10*

2015

Q3:

A conducting sphere of radius 5 cm has a total charge of 12 nC uniformly distributed on its surface in free space. Determine the displacement vector $\vec{D}$ on its surface and outside at a distance r from the centre of the sphere. *10*

2015

Q4:

With the help of a neat diagram, show that the potential due to a dipole at a point is given by $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p\cos\theta}{r^2}$, where p is the dipole moment of the charge distribution, θ is the angle between the line joining the centre of the dipole to the point of interest and the axis of the dipole. 10

2016

Q5:

Show that the displacement current between the plates of a parallel-plate capacitor is equal to the conduction current across the conductor. 10

2016

Q6:

A parallel-plate capacitor is connected to a 240 V AC supply having angular frequency of 300 rad/s. Find the r.m.s. value of the conduction current in the circuit. What is the displacement current between the plates of the capacitor? Given, $C = 200$ pF. 10

2016

Q7:

Discuss the principle of 'artificial dielectric'. Where do you find its use? 10

2017

Q8:

A charge $q = 2\ \mu C$ is placed at $a = 10$ cm from an infinite grounded conducting plane sheet. Find the *(i)* total charge induced on the sheet, *(ii)* force on the charge q and *(iii)* total work required to remove the charge slowly to an infinite distance from the plane. 10

2017

Q9:

Why does a soap bubble expand upon electrification?

A sphere of radius R contains a charge $+Q$ and a charge $-Q$ distributed uniformly in the upper and lower hemispheres respectively. Show that the dipole moment of charge distribution is $\frac{3}{4}QR\hat{k}$, where $\hat{k}$ is directed along the polar axis of the spherical coordinate system. 15

Q10:

Discuss briefly the features of 'guard rings'.

The plates of a capacitor are square-shaped, each of side l. The plates are inclined at an angle α to each other. The smallest distance between the plates is a. Calculate the capacitance when α is small. 15

Q11:

A 0·5 m long cylindrical medium between two conducting plates has uniform charge density of 100 nC/m^3. The axis of the cylindrical medium is along z-axis. The left plate is at z = 0 and has a potential of 10 kV and the right plate is grounded. Determine the electric field at axial distance z = 0·2 m. *15*

2018

Q12:

Starting from the expression for the electrostatic potential

$$\phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_V \frac{\rho(\vec{r_0})}{\left|\vec{r} - \vec{r_0}\right|} dV_0$$

obtain Poisson's equation $\nabla^2\phi = -\frac{\rho}{\varepsilon_0}$.

[Symbols have their usual meanings] *10*

2019

Q13:

Find the capacitance of two concentric spherical metal shells having radii a and b. *10*

2019

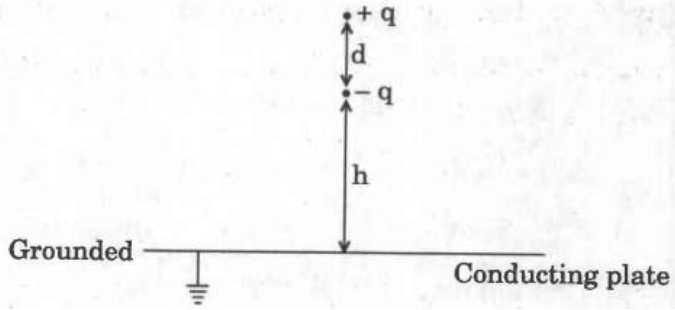
Q14:

Two conducting planes, intersecting at right-angles to each other, are kept at a potential ϕ_0. Calculate the potential at a point in space if the total charge on a plane of area α be Q. *15*

2019

Q15:

A vertically oriented electric dipole having dipole moment $\vec{p}$ is kept at height h above an infinitely large horizontal conducting plate, which is grounded as shown in the diagram. Calculate the force between the electric dipole and the conducting plate by using method of images. *10*



2020

Q16:

Write expressions for divergence and curl of an electrostatic field. From these, obtain Poisson and Laplace equations.

Two concentric conducting spherical shells having radii r_1 and r_2 ($r_1 < r_2$) are charged to potentials V_1 and V_2, respectively. What are the electric potential and hence electric field in the space between the shells ? Also find the charge on the inner shell. *15*

2020

Q17:

Given that the electric potential of a system of charges is $V = \frac{12}{r^2} + \frac{1}{r^3}$ volt. Calculate the electric field vector at the Cartesian point (4, 2, 3) m. *10*

2021

Q18:

A region 1, $z < 0$, has a dielectric material with $\varepsilon_r = 3{\cdot}2$ and a region 2, $z > 0$ has a dielectric material with $\varepsilon_r = 2{\cdot}0$. Let the displacement vector in the region 1 be, $\vec{D}_1 = -30\,a_x + 50\,a_y + 70\,a_z\ nCm^{-2}$. Assume the interface charge density is zero. Find in the region 2, the $\vec{D}_2$ and $\vec{P}_2$, where $\vec{P}_2$ is the electric polarization vector in the region 2. *20*

2021

Q19:

Given an infinite line charge of charge density 2 nCm^{-1} parallel to the y-axis and passing through the point (3, 0, 4) m and an infinite sheet of charge of charge density 4 nCm^{-2} parallel to the x-y plane and passing through the point (0, 0, 6) m. Calculate the electric field intensity at the point (10, 10, 10) m. Assume free space. *15*

2021

Q20:

Find the energy stored in a system of four charges $Q_1 = 1$ nC, $Q_2 = 2$ nC, $Q_3 = 3$ nC and $Q_4 = 4$ nC placed at the cartesian coordinates $R_1(1, 1)$, $R_2(2, 1)$, $R_3(1, 4)$ and $R_4(2, 2)$, respectively. Assume free space. *10*

2023

Q21:

A neutral atom consists of a point nucleus +q surrounded by a uniformly charged spherical cloud (–q) of radius r. Show that when such an atom is placed in a weak external electric field $\vec{E}$, the atomic polarizability of the atom is proportional to the volume of the sphere. *15*

2023

Q22:

A circular ring of radius R lying on the x-y plane and centred at the origin, carries a uniform line charge λ. Find the first three terms (monopole, dipole and quadrupole) of the multipole expansion of potential $V(r, \theta)$. *20*

2023

Q23:

Two charges $Q_1 = 3$ nC and $Q_2 = 4$ nC are placed at the cartesian points (0, 2, 2) m and (0, – 2, 4) m, respectively. The z = 0 plane is connected to the ground. Calculate the electric potential and the electric field at the point (3, 2, 4) m using the method of images. *15*

2023

Q24:

In spherical coordinates, $V = -25$ V on a conductor at $r = 2$ cm and $V = 150$ V on another conductor at $r = 35$ cm. The space between the conductors is a dielectric for which $\varepsilon_r = 3{\cdot}12$. Find the surface charge densities on the conductors. 10

2024

Q25.

The magnitude of the average electric field normally present in the Earth's atmosphere just above the surface of the Earth is about 150 N/C, directed radially inward, toward the centre of the Earth. What is the total net surface charge carried by the Earth? Assume the Earth to be a conductor. (The radius of the Earth is $6{\cdot}37 \times 10^6$ m) 10

2024

Q26.

Consider a point charge of 5 nC placed at a distance of 1 m from a perfect conducting plane (z = 0) of infinite extent. Find the electric field at a point (2, 2, 0) m and show that it is normal to the plane. *10*

2025

Q27.

Consider a conducting sphere of radius 'a' in a uniform electric field $\vec{E}$. Find the induced surface charge density on the sphere and determine the electric field $\vec{E}$ at a point P characterized by radius vector $\vec{r}$. *20*

2025