Tutorial 1:

- Wave-particle duality;
- Schrodinger equation and expectation values;
- Uncertainty principle

Q1:

The normalized wave function for the electron in the ground state of the hydrogen atom is given by $\psi(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-\frac{r}{a_0}}$,

where a_0 is the radius of the first Bohr orbit. Calculate the probability of finding the electron within a distance r_0 of the proton in the ground state. 10

How are electrons

2013

Q2:

In a series of experiments on the determination of the mass of a certain elementary particle, the results showed a variation of $\pm$ 20 m_e, where m_e is the electron mass. Estimate the lifetime of the particle. 10

2013

Q3:

Find the deBroglie wave length of (i) a neutron and (ii) an electron moving with a kinetic energy of 500 eV. (1 eV = 1.602×10^{-19} J) 10

2014

Q4:

The mean life time of Lambda (Λ^0) particle is 2.6×10^{-10} s. What will be the uncertainty in the determination of its mass in eV ? 10

2014

Q5:

Obtain an expression for the probability current for the plane wave $\psi(x, t) = \exp[i(kx - \omega t)]$. Interpret your result. 10

2015

Q6:

Write the time independent Schrödinger equation for a bouncing ball. 10

2015

Q7:

Normalized wave function of a particle is given below :

$$\psi = N \exp\left(\frac{-x^2}{2a^2} + ikx\right).$$

Find the expectation value of position. 10

2015

Q8:

A typical atomic radius is about 5×10^{-15} m and the energy of β-particle emitted from a nucleus is at most of the order of 1 MeV. Prove on the basis of uncertainty principle that the electrons are not present in nuclei. *10*

2016

Q9:

Using uncertainty principle, calculate the size and energy of the ground state of hydrogen atom. *10*

2016

Q10:

A beam of 4·0 keV electrons from a source is incident on a target 50·0 cm away. Find the radius of the electron beamspot due to Heisenberg's uncertainty principle. 10

2017

Q11:

Estimate the de Broglie wavelength of the electron orbitting in the first excited state of the hydrogen atom. 10

2017

Q12:

The wave function of a particle is given as $\psi(x)=\frac{1}{\sqrt{a}}e^{-|x|/a}$. Find the probability of locating the particle in the range $-a \le x \le a$. 10

2018

Q13:

Which of the following functions is/are acceptable solution(s) of the Schrödinger equation?

(i) $\psi(x) = Ae^{-ikx} + Be^{ikx}$

(ii) $\psi(x) = Ae^{-kx} + Be^{kx}$

(iii) $\psi(x) = A\sin 3kx + B\cos 5kx$

(iv) $\psi(x) = A\sin 3kx + B\sin 5kx$

(v) $\psi(x) = A\tan kx$

Explain your answer. 15

2018

Q14:

Show that the mass and linear momentum of a quantum mechanical particle can be given by $m = h/(\lambda v)$ and $p = h/\lambda$, respectively, where h, λ and v are Planck's constant, wavelength and velocity of the particle, respectively. Comment on the wave-particle duality from these relations. 10

2019

Q15:

State and express mathematically the three uncertainty principles of Heisenberg. Highlight the physical significance of these principles in the development of Quantum Mechanics. 10

2019

Q16:

Estimate the size of hydrogen atom and the ground state energy from the uncertainty principle. 15

2019

Q17:

A particle is described by the wave function $\psi(x) = \left(\frac{\pi}{4}\right)^{-1/4} e^{-\alpha x^2/2}$.. Calculate Δx and Δp for the particle, and verify the uncertainty relation $\Delta x \Delta p = \frac{\hbar}{2}$. 15

2020

Q18:

Using the uncertainty principle $\Delta x \Delta p \geq \frac{\hbar}{2}$, estimate the ground state energy of a harmonic oscillator. 15

2020

Q19:

Normalised wave function of hydrogen atom for $1s$ state is

$$\psi_{100} = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}, \text{ where } a_0 = \frac{\hbar^2}{me^2}$$

being the Bohr radius. Calculate the expectation value of potential energy in this state. 10

2021

Q20:

Find the uncertainty in the momentum of a particle when its position is determined within 0·02 cm. Find also the uncertainty in the velocity of an electron and α-particle respectively when they are located within 15×10^{-8} cm. 15

2021

Q21:

A particle of rest mass m_0 has a kinetic energy K, show that its de Broglie wavelength is given by

$$\lambda = \frac{hc}{\sqrt{[K(K+2m_0c^2)]}}$$

Hence calculate the wavelength of an electron of kinetic energy 2 MeV. What will be the value of λ if $K << m_0c^2$? 15

2021

Q22:

What is de Broglie concept of matter wave ? Evaluate de Broglie wavelength of Helium that is accelerated through 300 V.
(Given mass of proton = Mass of neutron = $1 \cdot 67 \times 10^{-27}$ kg) 10

2022

Q23.

A particle constrained to move along x-axis in the domain $0 \leqslant x \leqslant L$ has a wave function $\psi(x) = \sin\left(\frac{n\pi x}{L}\right)$, where n is an integer. Normalize the wave function and evaluate the expectation value of momentum of the particle. 15

2023

Q24.

A particle limited to the x-axis has the wave function $\phi(x) = bx^2$ between $x = 0$ and $x = 2$; the wave function $\phi(x) = 0$ elsewhere.

(i) Find the probability that the particle can be found between $x = 1{\cdot}0$ and $x = 1{\cdot}5$.

(ii) Find the expectation value $< x >$ of the particle position. *10*

2024

Q25.

Explain how the uncertainty in position is different from the uncertainty or inaccuracy of the measuring instruments. 10

2025