Tutorial 2:

- Motion under a central force, Keppler's laws
- Fields and potentials; Gravitational field and potential due to spherical bodies,
- Gauss and Poisson equations, gravitational self-energy;
- Two-body problem; Reduced mass;
- Rutherford scattering; Centre of mass and laboratory reference frames.

Q1.

A particle describes a circular orbit under the influence of an attractive central force directed towards a point on the circle. Show that the force varies as the inverse fifth power of distance. *15*

2013

Q2.

The density inside a solid sphere of radius a is given by $\rho = \frac{\rho_0 a}{r}$, where ρ_0 is the density at the surface and r denotes the distance from the centre. Find the gravitational field due to this sphere at a distance 2a from its centre. *10*

Q4:

Discuss the problem of scattering of charged particle by a coulomb field. Hence, obtain an expression for Rutherford scattering cross-section. What is the importance of the above expression ? *25*

2014

Q5:

A charged particle is moving under the influence of a point nucleus. Show that the orbit of the particle is an ellipse. Find out the time period of the motion. *15*

2014

Q6:

Draw a neat diagram to explain the scattering of an incident beam of particles by a centre of force. *10*

2015

Q7:

Show that the differential scattering cross-section can be expressed as

$$\sigma(\theta) = \frac{s}{\sin\theta}\left|\frac{ds}{d\theta}\right|,$$

where s is the impact parameter and θ is the scattering angle. *15*

2015

Q8:

A body moving in an inverse square attractive field traverses on elliptical orbit with eccentricity *e* and period γ. Find the time taken by the body to traverse the half of the orbit that is nearer the centre of force. Explain briefly why a comet spends only 18% of its time on the half of its orbit that is nearer the sun. 10

2016

Q9:

Express angular momentum in terms of kinetic, potential and total energy of a satellite of mass *m* in a circular orbit of radius *r*. 10

2017

Q10:

A diatomic molecule can be considered to be made up of two masses m_1 and m_2 separated by a fixed distance *r*. Derive a formula for the distance of centre of mass, *C*, from mass m_1. Also show that the moment of inertia about an axis through *C* and perpendicular to *r* is μr^2, where $\mu = \frac{m_1 m_2}{m_1 + m_2}$. 15

2017

Q11:

If a particle of mass m is in a central force field $f(r)\hat{r}$, then show that its path must be a plane curve, where $\hat{r}$ is a unit vector in the direction of position vector $\vec{r}$. *10*

2018

Q12:

Use Gauss's theorem to calculate the gravitational potential due to a solid sphere at a point outside the sphere. Calculate the amount of work required to send a body of mass m from the Earth's surface to a height R/2, where R is the radius of the Earth. *15*

2018

Q13:

(i) केन्द्रीय बल क्या है ? केन्द्रीय बल के दो उदाहरण दीजिए ।
What is a central force ? Give two examples of the central force.

(ii) दिखाइए कि केन्द्रीय बल क्षेत्र में एक कण का कोणीय संवेग ($\vec{L}$) गति का स्थिरांक है ।
Show that the angular momentum ($\vec{L}$) of the particle in a central force field is a constant of motion. *10*

2019

Q14:

Show that the cross-section for elastic scattering of a point particle from an infinitely massive sphere of radius R is $\frac{R^2}{4}$. What is the inference of this result ? *10*

2019

Q15:

A particle moving in a central force field describes the path $r = ke^{\alpha\theta}$, where k and α are constants. If the mass of the particle is m, find the law of force. *10*

2021

Q16:

The radius of the Earth is $6{\cdot}4 \times 10^6$ m, its mean density is $5{\cdot}5 \times 10^3$ kg/m^3 and the universal gravitational constant is $6{\cdot}66 \times 10^{-11}$ Nm^2/kg^2. Calculate the gravitational potential on the surface of the Earth. *10*

2021

Q17:

An electron is moving under the influence of a point nucleus of atomic number Z. Show that the orbit of the electron is an ellipse. 10

2022

Q18:

Calculate the gravitational self-energy of the Earth.

Given :

Mass of Earth $M_e = 6 \times 10^{24}$ kg and the Radius of Earth $R_e = 6{\cdot}4 \times 10^6$ m *10*

2023

Q19:

(i) Derive the expressions for gravitational potentials at a point

(I) outside the spherical shell,

(II) inside the spherical shell. *10*

(ii) Calculate the escape velocity of a body of mass 10 kg from the surface of Moon $\left(g_{Moon} = \frac{1}{6} g_{Earth}\right)$.

Mass of Moon = $7{\cdot}3 \times 10^{22}$ kg

Radius of Moon = $1{\cdot}7 \times 10^6$ m *10*

2023

Q20.

2. *(a)* *(i)* ग्रहीय गति के केप्लर के नियमों की संक्षेप में चर्चा कीजिये।

Briefly discuss the Kepler's laws of planetary motion. 5

(ii) दर्शाइये कि पृथ्वी की सतह पर पलायन वेग $V_e = \sqrt{2gR}$ है, जहाँ $g = 9{\cdot}8\ \text{m/s}^2$ और R पृथ्वी की त्रिज्या है।

Show that the escape velocity V_e on the surface of the Earth is given by $V_e = \sqrt{2gR}$, where $g = 9{\cdot}8\ \text{m/s}^2$ and R is the radius of the Earth. 5

(iii) समान द्रव्यमान के दो उपग्रह A और B पृथ्वी के चारों ओर क्रमशः उच्चताओं R और $5R$ पर परिक्रमण कर रहे हैं, जहाँ R पृथ्वी की त्रिज्या है। उनकी कक्षाओं को वृत्तीय मानकर उनकी गतिज ऊर्जाओं और स्थितिज ऊर्जाओं के अनुपातों की गणना कीजिये।

Two satellites A and B of same mass are orbiting the Earth at altitudes R and $5R$, respectively, where R is the radius of the Earth. Assuming their orbits to be circular, calculate the ratios of their kinetic and potential energies. 5

2024

Q21.

Derive the expression for the gravitational self-energy of a uniform solid sphere of mass M and radius R. *10*

2025