Tutorial 2:

- Solutions of the one-dimensional Schrodinger equation for a free particle (Gaussian wave-packet),
- particle in a box, particle in a finite well
- Linear harmonic oscillator
- Reflection and transmission by a step potential, by a rectangular barrier
- Particle in a three dimensional box, density of states, free electron theory of metals

Section A- Free Particle, Particle in a box, Particle in a finite well

Q1:

Obtain the time-dependent Schrödinger equation for a particle. Hence deduce the time-independent Schrödinger equation. 20

2014

Q2:

Solve the Schrödinger equation for a particle of mass m confined in a one-dimensional potential well of the form :

$$V = 0, \text{ when } 0 \le x \le L$$
$$= \infty, \text{ when } x < 0, x > L$$

Obtain the discrete energy values and the normalized eigen functions. 20

2014

Q3:

An electron is moving in a one dimensional box of infinite height and width 1 Å. Find the minimum energy of electron. 10

2014

Q4:

A particle trapped in an infinitely deep square well of width a has a wave function $\psi = \left(\frac{2}{\pi}\right)^{1/2} \sin\left(\frac{\pi x}{a}\right)$. The walls are suddenly separated by infinite distance. Find the probability of the particle having momentum between p and p+dp. **10**

2015

Q5:

An electron is confined to move between two rigid walls separated by 10^{-9} m. Compute the de Broglie wavelengths representing the first three allowed energy states of the electron and the corresponding energies. *10*

2016

Q6:

Calculate the lowest energy of an electron confined to move in a 1-dimensional potential well of width 10 nm. 10

2017

Q7:

Find the probability current density for the wave function

$$\psi(x, t) = [Ae^{ipx/\hbar} + Be^{-ipx/\hbar}]e^{-ip^2t/2m\hbar}$$

Interpret the result physically. 10

2020

Q8:

A particle is moving in a one dimensional box of width 50Å and infinite height. Calculate the probability of finding the particle within an interval of 15Å at the centres of the box when it is in its state of least energy. 15

2021

Q9:

An electron in a one-dimensional infinite potential well, defined by
$V(x) = 0$ for $-a \leqslant x \leqslant a$ and $V(x) = \infty$
otherwise, goes from $n = 4$ to $n = 2$ level and emits photon of frequency $3{\cdot}43\times10^{14}$ Hz. Calculate the width of the well. (Assume Plank's constant $h = 6{\cdot}626\times10^{-34}$ J.S. and mass of electron $m = 9{\cdot}11\times10^{-31}$ kg) 10

2022

Q10:

Calculate the zero point energy for a particle in an infinite potential well for the following cases :

(i) a 100 g ball confined on a 5 m long line.

(ii) an oxygen atom confined to a 2×10^{-10} m lattice.

(iii) an electron confined to a 10^{-10} m atom.

Why zero point energy is not important for macroscopic objects ? Comment. 10

2023

Q11.

Consider the potential $V(x) = \begin{cases} 0, & 0 < x < a \\ \infty, & \text{elsewhere} \end{cases}$

(a) Estimate the energies of the ground state as well as those of the first and the second excited states for

(i) an electron enclosed in a box of size $a = 10^{-10}$ m.

(ii) a 1 g metallic sphere which is moving in a box of size $a = 10$ cm.

(b) Discuss the importance of the Quantum effects for both of these systems.

(c) Estimate the velocities of the electron and the metallic sphere using uncertainty principle. 20

2023

Q12.

(i) Assuming the potential seen by a neutron in a nucleus to be schematically represented by a one-dimensional, infinite rigid wall potential of length 10^{-15} m, estimate the minimum kinetic energy of the electron.

(ii) Estimate the minimum kinetic energy of neutron bound within the nucleus as described above. Can an electron be confined in a nucleus ? Explain. *15*

2024

Q13.

Determine the ground state energy of an electron in an infinite potential well of width of 2 Å. 10

2025

Section B - Linear Harmonic Oscillator

Q1:

Using Schrödinger equation, obtain the eigenfunctions and eigenvalues of energy for a 1-dimensional harmonic oscillator. Sketch the profiles of eigenfunctions for first three energy states. 20

2017

Q2:

Calculate the zero-point energy of a system consisting of a mass of 10^{-3} kg connected to a fixed point by a spring which is stretched by 10^{-2} m by a force of 10^{-1} N. The system is constrained to move only in one direction. 10

2018

Q3:

Calculate the zero-point energy of a system consisting of a mass of 10^{-3} kg connected to a fixed point by a spring which is stretched by 10^{-2} m by a force of 10^{-1} N. The system is constrained to move only in one direction. 10

2018

Q4:

For a free quantum mechanical particle under the influence of a one-dimensional potential, show that the energy is quantized in discrete fashion. How do these energy values differ from those of a linear harmonic oscillator? 10

2019

Q5:

Write down the Hamiltonian operator for a linear harmonic oscillator. Show that the energy eigenvalue of the same can be given by $E_n = \left(n + \frac{1}{2}\right)\hbar\omega_0$ at energy state n with ω_0 being the natural frequency of vibration of the linear oscillator. Prove that $n = 0$ energy state has a wave function of typical Gaussian form. 15

2019

Q6:

Calculate the probability of finding a simple harmonic oscillator within the classical limits if the oscillator is in its normal state. Also show that if the oscillator is in its normal state, then the probability of finding the particle outside the classical limits is approximately 16%. 15

2021

Q7:

Consider a particle of mass m and charge q moving under the influence of a one dimensional harmonic oscillator potential. Assume it is placed in a constant electric field E. The Hamiltonian of this particle is therefore given by $H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 X^2 - qEX$. Obtain the energy expression and the wave function of the nth excited state of the particle. 10

2023

Q8.

Find the energy of the particle of mass m moving in a potential field $V(x) = \frac{2\hbar^2 b^2 x^2}{m}$ for which the time independent wave function is $\psi(x) = \exp(-bx^2)$. Here b is a constant. *10*

2024

Q9.

The ground state wave function of a harmonic oscillator is

$$\psi_0(x) = \left(\frac{m\omega}{\hbar\pi}\right)^{1/4} \exp\left(-\frac{m\omega x^2}{2\hbar}\right).$$

(i) At which point is the probability density maximum ?

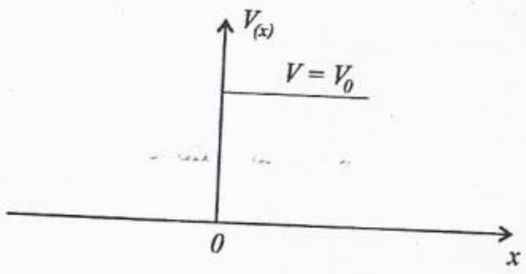
(ii) What is the value of the maximum probability density ? *15*

2024

Section C - Step Potential, Rectangular Barrier

Q1:

(a)



Consider a beam of particles incident on a one-dimensional step function potential with energy $E>V_0$ as shown in the above figure. Solve the Schrödinger equation and obtain expressions for the reflection and transmission coefficients. 30

(b) What are the limits of the reflection coefficient for $E \to V_0$ and $E \to \infty$? 10

(c) Discuss the cases $0 < E < V_0$ and $E < 0$. 10

2013

Q2:

Solve the Schrödinger equation for a step potential and calculate the transmission and reflection coefficient for the case when the kinetic energy of the particle E_0 is greater than the potential energy V (i.e., $E_0 > V$). *20*

2016

Q3:

Calculate the probability of transmission of an electron of 1·0 eV energy through a potential barrier of 4·0 eV and 0·1 nm width. 10

2017

Q4:

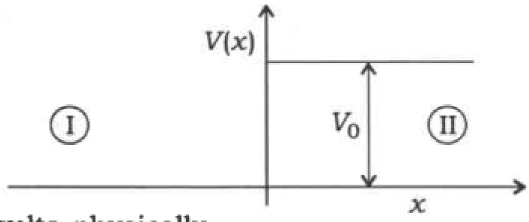
A beam of particles of energy 9 eV is incident on a potential step 8 eV high from the left. What percentage of particles will reflect back? 15

2018

Q5:

Write the wave functions for a particle on both sides of a step potential, for $E > V_0$:

$$V(x) = \begin{cases} V_0, & x > 0 \\ 0, & x < 0 \end{cases}$$



Interpret the results physically. 10

2020

Q6:

A beam of 12 eV electron is incident on a potential barrier of height 25 eV and width 0·05 nm. Calculate the transmission coefficient. 10

2021 (The question had an attached hyperbolic function table)

EXPONENTIAL AND HYPERBOLIC FUNCTIONS

x	e^x	e^{-x}	sinh x	cosh x	x	e^x	e^{-x}	sinh x	cosh x
·02	1·0202	·9802	·0200	1·0002	1·0	2·7183	·3679	1·1752	1·5431
·04	1·0408	·9608	·0400	1·0008	1·1	3·0042	·3329	1·3356	1·6685
·06	1·0618	·9418	·0600	1·0018	1·2	3·3201	·3012	1·5095	1·8107
·08	1·0833	·9231	·0801	1·0032	1·3	3·6693	·2725	1·6984	1·9709
·10	1·1052	·9048	·1002	1·0050	1·4	4·0552	·2466	1·9043	2·1509
·11	1·1163	·8958	·1102	1·0061	1·5	4·4817	·2231	2·1293	2·3524
·12	1·1275	·8869	·1203	1·0072	1·6	4·9530	·2019	2·3756	2·5775
·13	1·1388	·8781	·1304	1·0085	1·7	5·4739	·1827	2·6456	2·8283
·14	1·1503	·8694	·1405	1·0098	1·8	6·0497	·1653	2·9422	3·1075
·15	1·1618	·8607	·1506	1·0113	1·9	6·6859	·1496	3·2682	3·4177
·16	1·1735	·8521	·1607	1·0128	2·0	7·3891	·1353	3·6269	3·7622
·17	1·1853	·8437	·1708	1·0145	2·1	8·1662	·1225	4·0219	4·1443
·18	1·1972	·8353	·1810	1·0162	2·2	9·0250	·1108	4·4571	4·5679
·19	1·2092	·8270	·1911	1·0181	2·3	9·9742	·1003	4·9370	5·0372
·20	1·2214	·8187	·2013	1·0201	2·4	11·023	·0907	5·4662	5·5569
·21	1·2337	·8106	·2115	1·0221	2·5	12·182	·0821	6·0502	6·1323
·22	1·2461	·8025	·2218	1·0243	2·6	13·464	·0743	6·6947	6·7690
·23	1·2586	·7945	·2320	1·0266	2·7	14·880	·0672	7·4063	7·4735
·24	1·2712	·7866	·2423	1·0289	2·8	16·445	·0608	8·1919	8·2527
·25	1·2840	·7788	·2526	1·0314	2·9	18·174	·0550	9·0596	9·1146
·26	1·2969	·7711	·2629	1·0340	3·0	20·085	·0498	10·018	10·068
·27	1·3100	·7634	·2733	1·0367	3·1	22·198	·0450	11·076	11·121
·28	1·3231	·7558	·2837	1·0395	3·2	24·532	·0408	12·246	12·287
·29	1·3364	·7483	·2941	1·0423	3·3	27·113	·0369	13·538	13·575
·30	1·3499	·7408	·3045	1·0453	3·4	29·964	·0334	14·965	14·999
·31	1·3634	·7335	·3150	1·0484	3·5	33·115	·0302	16·543	16·573
·32	1·3771	·7261	·3255	1·0516	3·6	36·598	·0273	18·285	18·313
·33	1·3910	·7189	·3360	1·0550	3·7	40·447	·0247	20·211	20·236
·34	1·4050	·7118	·3466	1·0584	3·8	44·701	·0224	22·339	22·362
·35	1·4191	·7047	·3572	1·0619	3·9	49·402	·0202	24·691	24·711
·36	1·4333	·6977	·3678	1·0655	4·0	54·598	·0183	27·290	27·308
·37	1·4477	·6907	·3785	1·0692	4·1	60·340	·0166	30·162	30·178
·38	1·4623	·6839	·3892	1·0731	4·2	66·686	·0150	33·336	33·351
·39	1·4770	·6771	·4000	1·0770	4·3	73·700	·0136	36·843	36·857
·40	1·4918	·6703	·4107	1·0811	4·4	81·451	·0123	40·719	40·732
·41	1·5068	·6636	·4216	1·0852	4·5	90·017	·0111	45·003	45·014
·42	1·5220	·6570	·4325	1·0895	4·6	99·484	·0100	49·737	49·747
·43	1·5373	·6505	·4434	1·0939	4·7	109·95	·00910	54·969	54·978
·44	1·5527	·6440	·4543	1·0984	4·8	121·51	·00823	60·751	60·759
·45	1·5683	·6376	·4653	1·1030	4·9	134·29	·00745	67·141	67·149
·46	1·5841	·6313	·4764	1·1077	5·0	148·41	·00674	74·203	74·210
·47	1·6000	·6250	·4875	1·1125	5·1	164·02	·00610	82·008	82·014
·48	1·6161	·6188	·4986	1·1174	5·2	181·27	·00552	90·633	90·639
·49	1·6323	·6126	·5098	1·1225	5·3	200·34	·00499	100·17	100·17
·50	1·6487	·6065	·5211	1·1276	5·4	221·41	·00452	110·70	110·71
·60	1·8221	·5488	·6367	1·1855	5·5	244·69	·00409	122·34	122·35
·70	2·0138	·4966	·7586	1·2552	5·6	270·43	·00370	135·21	135·21
·80	2·2255	·4493	·8881	1·3374	5·7	298·87	·00335	149·43	149·43
·90	2·4596	·4066	1·0265	1·4331	5·8	330·30	·00303	165·15	165·15
					5·9	365·04	·00274	182·52	182·52
					6·0	403·43	·00248	201·71	201·72

Q7:

Set up the Schrodinger's wave equation for one dimensional potential barrier and obtain the probability of tunneling. 20

2022

Q8:

Consider a stream of particles of mass m each moving in the positive x-direction with kinetic energy E towards the potential barrier

$$V(x) = 0 \quad \text{for} \quad x \leqslant 0$$

$$V(x) = \frac{3E}{4} \quad \text{for} \quad x > 0$$

Find the fraction of particles reflected at $x = 0$. 15

2023

Q9.

Obtain the expressions for reflection coefficient (R) and transmission coefficient (T) for reflected waves and transmitted waves from an infinite thin barrier. 15

2025

Q10.

For a potential with the boundary conditions

$$V(x) = \begin{cases} 0, & x < -a \\ V, & -a < x < a \\ 0, & x > a \end{cases}$$

solve the Schrödinger's equation in one dimension and find out the conditions for tunnelling. 15

2025

Section D - 3D box, density of states, free electron theory of metals

Q1:

Solve the Schrödinger equation for a particle in a three dimensional rectangular potential barrier. Explain the terms degenerate and non-degenerate states in this context. 30

2015

Q2:

Calculate the density of states for an electron moving freely inside a metal with the help of quantum mechanical Schrödinger's equation for free particle in a box. *10*

2016

Q3:

Show that for free electron gas, the density of states in three dimensions (3D) varies as $E^{1/2}$, and this dependence changes to E^0 for 2D (quantum well), $E^{-1/2}$ for 1D (quantum wire) and δ function for 0D (quantum dot). 15

2018

Q4:

How do you define density of states? Show that the density of states with wave vector less than $\vec{k}$ in a three-dimensional cubic box of volume V can be given by

$$D(\omega) = \frac{V}{2\pi^2} k^2 \left(\frac{dk}{d\omega}\right)$$

in the frequency spectrum between ω and $\omega + d\omega$. Here, assume that the number of modes per unit range of k is $L/(2\pi)$, L being the length of each side of the cubic box. 20

2019

Q5:

(i) Show that for a given principal quantum number n, there are n^2 possible states of the atom.

(7 Marks) 2022

Q6.

(i) Using free electron theory of metals, calculate the Fermi energy level of sodium atom at absolute zero. Assume that sodium has one free electron per atom and its density is $0{\cdot}97$ gm/cm^3.

(ii) Draw the energy level diagram and mathematical expressions for the following :

I. E_n of an electron confined in a one-dimensional box

II. Linear harmonic oscillator

Make a qualitative comparison of the above two cases. *10+10=20*

2024

Q7.

What is the density of states? For a relativistic particle of rest mass μ, prove that the density of states in the extreme relativistic limit $(E \gg \mu c^2)$

$$g(E) = \frac{V}{\pi^2 \hbar^3 c^3} E^2$$

where $g(E)$ = density of states, V = volume of the system containing the particle, E = total energy, c = velocity of light and h = Planck's constant. 20

2025