Tutorial 3: Mechanics of Rigid Bodies:

- System of particles; Centre of mass, angular momentum, equations of motion
- Conservation theorems for energy, momentum and angular momentum;
- Elastic and inelastic collisions; Rigid body; Degrees of freedom, Euler's theorem,
- angular velocity, angular momentum, moments of inertia, theorems of parallel and perpendicular axes, equation of motion for rotation;
- Molecular rotations (as rigid bodies); Di and tri-atomic molecules;
- Precessional motion; top, gyroscope.

Q1.

10×5=50

Show that the kinetic energy and angular momentum of the torque free motion of a rigid body is constant. *10*

2013

Q2.

If I' and I be the Moments of Inertia of a body about an axis passing through an arbitrary origin and about a parallel axis through the centre of mass respectively, show that $I' = MR^2 + I$, where $\vec{R}$ is the position vector of the centre of mass with respect to the arbitrary origin and M is the mass of the body. *10*

2014

Q3.

Consider a rigid body rotating about an axis passing through a fixed point in the body with an angular velocity $\vec{\omega}$. Determine the kinetic energy of such a rotating body in a coordinate system of principal axis. If the Earth suddenly stops rotating, what will happen to the rotational kinetic energy ? Comment in detail. *25*

2014

Q4.

A body turns about a fixed point. Show that the angle between its angular velocity vector and its angular momentum vector about a fixed point is always acute. *15*

2014

Q5:

Show that the moment of inertia of a circular disc of mass M and radius R about an axis passing through its centre and perpendicular to its plane is $\frac{1}{2}MR^2$. *15*

2015

Q6:

(i) कार्बन मोनो-ऑक्साइड (CO) गैस के अणु में कार्बन तथा ऑक्सीजन के परमाणुओं के केन्द्रों के बीच की दूरी $1 \cdot 130 \times 10^{-10}$ m है। कार्बन परमाणु के सापेक्ष अणु के संहति-केन्द्र का परिकलन करें।

The distance between the centres of the carbon and oxygen atoms in the carbon monoxide (CO) gas molecule is $1 \cdot 130 \times 10^{-10}$ m. Locate the centre of mass of the molecule relative to the carbon atom.

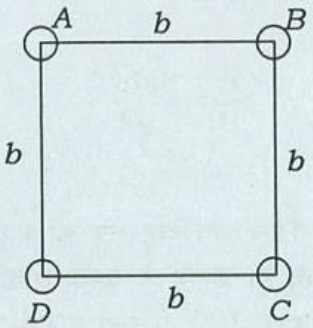
(ii) त्रिज्या a के समांगी अर्धवृत्ताकार प्लेट का संहति-केन्द्र ज्ञात करें।

Find the centre of mass of a homogeneous semicircular plate of radius a. 10

2016

Q7:

Four solid spheres A, B, C and D, each of mass m and radius a, are placed with their centres on the four corners of a square of side b as shown in the figure below :



Calculate the moment of inertia of the system about one side of the square. Also calculate the moment of inertia of the system about a diagonal of the square. 20

2016

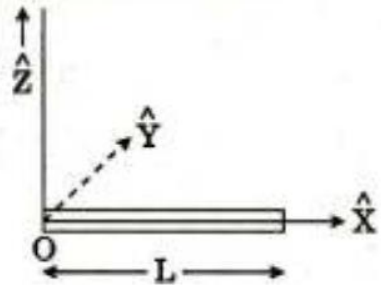
Q8:

Express angular momentum in terms of kinetic, potential and total energy of a satellite of mass m in a circular orbit of radius r. 10

2017

Q9:

A rod of length L has non-uniform linear mass density (mass per unit length) λ, which varies as $\lambda = \lambda_0 \left(\frac{S}{L}\right)$; where λ_0 is a constant and S is the distance from the end marked 'O' (as shown in the figure). Find the centre of mass of the rod. *15*



2018

Q10:

(i) दृढ़ द्विपरमाणुक अणु का विभिन्न सममिति अक्षों पर द्रव्यमान/संहति-केन्द्र के सापेक्ष जड़त्व आघूर्ण ज्ञात कीजिए ।

Find the moments of inertia of rigid diatomic molecule about different axes of symmetry through the centre of mass.

(ii) एक प्रोटॉन एक इलेक्ट्रॉन से 1837 गुना भारी है । हाइड्रोजन परमाणु का द्रव्यमान/संहति-केन्द्र ज्ञात कीजिए ।

A proton is 1837 times heavier than an electron. Find the centre of mass of hydrogen atom. *15*

2019

Q11:

Write down Euler's dynamical equations of motion (no derivation) of a rigid body about a fixed point under the action of a torque. Show that the kinetic energy of the torque-free motion is constant. *10*

2019

Q12:

Where do you find the applications of gyroscope ?

A top of mass 0·200 kg is made up of a thin disc of radius 0·12 m. It is pierced in the centre and a pin of negligible mass is mounted normal to its plane. The pivot under the disc is 0·03 m long. The top is made to spin with its axis making an angle $\theta = 20°$ with the vertical and a precessional angular speed of 2 rad/s. Calculate the angular speed with which it spins. *15*

2019

Q13:

Determine the location of the centre of mass of a uniform solid hemisphere of radius R and mass M from the centre of its base. *10*

2020

Q14:

Obtain expressions for the moment of inertia of a solid cone about its (i) vertical axis and (ii) axis passing through the vertex and parallel to its base. *20*

2020

Q15:

A body of mass m at rest splits into two masses m_1 and m_2 by an explosion. After the split the bodies move with a total kinetic energy T in opposite direction. Show that their relative speed is $\sqrt{2Tm/m_1m_2}$. *15*

2021

Q16:

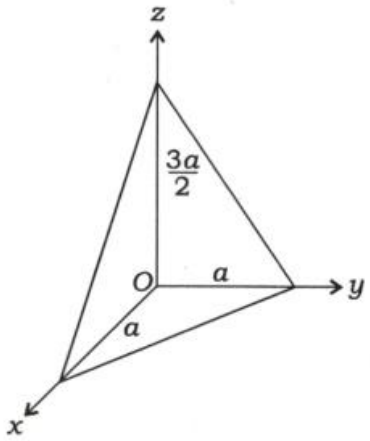
(i) Define moment of inertia and radius of gyration of a body of mass M rotating about an axis. State and prove Parallel Axis theorem on moment of inertia.

(ii) A sphere of mass 0·5 kg rolls on a smooth surface without slipping with a constant velocity of 3·0 m/s. Calculate its total kinetic energy. *15+5*

2021

Q17:

A homogeneous right triangular pyramid with the base side a and height $\frac{3a}{2}$ is shown below. Obtain the moment of inertia tensor of the pyramid :



20

2022

Q18:

A particle P of mass m_1 collides with another particle Q of mass m_2 at rest. The particles P and Q travel at angles θ and ϕ, respectively, with respect to the initial direction of P. Derive the expression for the maximum value of θ. 15

2022

Q19:

(i) The quantities of rotatory motion are analogous to those of translatory motion. Write the corresponding equations of translatory and rotatory motion. *5*

(ii) Describe the theorems of perpendicular and parallel axes in case of a plane lamina. *10*

2023

Q20:

(i) Prove that the separation of two colliding particles is same, when observed in centre of mass and laboratory systems. *10*

(ii) Determine the kinetic energy of a thin disc of mass 0·5 kg and radius 0·2 m rotating with 100 rotations per second around the axis passing through its centre and perpendicular to its plane. *5*

2023

Q21.

Show that the angular momentum of a rigid body consisting of n particles of masses m_i, $i = 1, 2, 3, \cdots, n$, rotating with an instantaneous angular velocity ω about an axis passing through the origin O of the coordinate system $OXYZ$ is given by $L = I \cdot \omega$, where I is known as the inertia tensor. 20

2024

Q22.

Consider a large stationary cylinder of inner radius R. A smaller solid cylinder of radius r rolls without slipping inside the larger cylinder. Determine the equation of motion of the smaller cylinder. *10*

2025

Q23.

A body moves about a point 'O' under no force, the principal moments of inertia at 'O' being 3A, 5A and 6A. The components of the initial angular velocity about the principal axes are $\omega_1 = n$, $\omega_2 = 0$ and $\omega_3 = n$. Find the components ω_1, ω_2 and ω_3 for large values of time t. *20*

2025

Q24.

A cube of mass M and side 'a' is rotating with angular velocity ω around one of its edges, which is, say, along the x-axis. Obtain the expressions for its angular momentum and kinetic energy. *20*

(Given that the $I_{XX} = \frac{2}{3} Ma^2$, $I_{YX} = -\frac{1}{4} Ma^2$ and $I_{ZX} = -\frac{1}{4} Ma^2$)

2025