Tutorial 4

- Macro and micro states,
- statistical distributions,
- Maxwell-Boltzmann
- Bose-Einstein and Fermi-Dirac distributions,
- applications to specific heat of gases and blackbody radiation;
- Concept of negative temperatures.

Q1:

N particles obeying Classical Statistics are distributed among three states having energies $\varepsilon_1 = 0$, $\varepsilon_2 = k_B T$ and $\varepsilon_3 = 2\ k_B T$, where k_B is Boltzmann constant. If the total equilibrium energy of the system is $1000\ k_B T$, calculate the value of N. *10*

2013

Q2:

In deriving radiation laws, we consider a cubical container of volume V containing a photon gas in equilibrium. Calculate the differential number of allowed normal modes of frequency ω. *10*

2014

Q3:

Show that both Fermi-Dirac and Bose-Einstein distribution functions at an energy E are given by :

$$f(E) \simeq \exp[(\mu - E) / k_B T],$$

where f(E) is much smaller than unity, μ and $k_B T$ are the chemical potential and thermal energy of the atom. *10*

2014

Q4:

Using Maxwell-Boltzmann distribution law prove that there cannot be any negative absolute temperature. *10*

2014

Q5:

Consider a system of N particles and a phase space consisting of only two states with energies 0 and ε (> 0). Obtain the expressions for the partition function and the internal energy of the system, if it obeys M-B statistics. 10

2016

Q6:

Using the Maxwell-Boltzmann distribution law, show that there cannot be any negative absolute temperature. 10

2017

Q7:

Explain Bose-Einstein distribution and obtain the same from the grand canonical ensemble. 15

2017

Q8:

A system having two energy levels, $-\frac{1}{2}\Delta$ and $+\frac{1}{2}\Delta$ with $\Delta = 10$ meV is populated by 1000 particles at a low temperature close to 100 K. Obtain the average energy per particle using classical distribution law. ***15***

2018

Q9:

Schematically, show the variation of density of states, $D(\varepsilon)$ and distribution function, $f(\varepsilon, T)$, of particles in a non-relativistic Fermi gas at high temperatures.

At a temperature T, an electron occupies a state with energy 100 meV above the Fermi energy (ε_F) with the probability of 1%. Find the temperature T. ***20***

2018

Q10:

A gas has only two particles, a and b. With the help of a diagram, show that how these two particles can be arranged in the three quantum series 1, 2, 3 using (i) Maxwell-Boltzmann, (ii) Fermi-Dirac, and (iii) Bose-Einstein statistics. *10*

2019

Q11:

If the partition function for a perfect gas is given by

$$Z = \frac{V}{h^3}(2\pi\, mkT)^{3/2}$$

calculate (i) average kinetic energy per molecule and (ii) specific heat of the gas. *15*

2019

Q12:

The energy level of a quantum harmonic oscillator with frequency ν is given by

$$E_n = \left(n + \frac{1}{2}\right) h\nu, \text{ where } n = 0, 1, 2, \ldots$$

Calculate its partition function.

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Calculate the partition function of a two level system. *10*

2020

Q13:

Eight indistinguishable balls are to be arranged in six distinguishable boxes. Calculate the total number of ways in which the above can be done. *10*

2021

Q14:

(i) Write the expression for the Fermi-Dirac distribution. Plot the Fermi-Dirac distribution at $T = 0$ and for $T_1 > T_2 > 0$. Now from the plot propose two alternative definitions of the Fermi level.

(ii) Calculate the probability of an electron occupying an energy level 0·02 eV above the Fermi level at $T = 300$ K. *15+5*

2021

Q15:

What do you understand by negative temperature? Write and explain various restrictions on a system for the concept of negative temperature to be meaningful. 15

2022

Q16:

Write down the expressions for the Fermi-Dirac distribution and the Bose-Einstein distribution. Plot the distributions as a function of the energy. *10*

2023

Q17.

What do you understand by macrostates and microstates? Briefly explain. 5

2024

Q18.

Calculate the Fermi energy in electron-volt for sodium assuming that it has one free electron per atom. The density of sodium = 0·97 gm/cc and the atomic weight of sodium is 23. 10

2024

Q19.

A three-level laser system emits laser light at a wavelength of 550 nm. If the population of the upper level exceeds that of the lower level by 25%, determine the negative temperature characterizing the system. 10

2024

Q20.

Consider a mixture of N_A molecules of a monatomic gas A and N_B molecules of a monatomic gas B. For this mixture, obtain the Helmholtz free energy and pressure. (The particle partition function for a monatomic gas is $q = \left(\frac{2\pi mkT}{h^2}\right)^{\frac{3}{2}} V$). *15*

2025

Q21.

Derive the Planck's radiation law for blackbody radiation using the Bose-Einstein distribution function. Explain how results from quantum statistics differ from classical results derived from the Rayleigh-Jeans law. *20*

2025