Tutorial 5: Special Relativity:

- Michelson-Morley experiment and its implications;
- Lorentz transformations-length contraction, time dilation, addition of relativistic velocities, aberration and Doppler effect, mass-energy relation, simple applications to a decay process;
- Four dimensional momentum vector; Covariance of equations of physics.

Q1.

Show that a particle of rest mass m_0, total energy E and linear momentum $\vec{p}$ satisfies the relation

$$E^2 = c^2p^2 + m_0^2c^4$$

where c is the velocity of light in free space. *10*

2013

Q2.

(d) Derive the relativistic length contraction using Lorentz transformation. *10*

2013

Q3.

A particle of rest mass $M = 4 \times 10^{-27}$ kg, disintegrates into two particles of rest masses $M_1 = 3 \times 10^{-27}$ kg and $M_2 = 1 \times 10^{-27}$ kg. Show that the energies E_1 and E_2 of these two parts after disintegration satisfy the condition $E_1 = 3\,E_2$ while moving in opposite directions with equal linear momenta. Give necessary mathematical derivation. *15*

2013

Q4.

Show that the operator $\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)$ is invariant under Lorentz transformations. *20*

2013

Q5:

A mirror is moving through vacuum with a relativistic speed v in the x-direction. A beam of light with frequency ω_i is normally incident (from $x = \infty$) on the mirror.

(i) What is the frequency of the reflected light expressed in terms of ω_i, c and v ?

(ii) What is the energy of each reflected photon ? *25*

In question 3(a), if the average energy flux of the incident beam is P_i (watts/m^2), what is the average energy flux of the reflected beam ? *15*

Read above question

2014

Q6:

Prove mathematically that the addition of any velocity of a particle to the velocity of light in free space merely reproduces the velocity of light in free space only. *10*

2015

Q7:

Show that the rest mass energy of an electron is 0·51 MeV (use the standard values of the physical parameters). *10*

2015

Q8:

Calculate the percentage contraction in the length of a rod in a frame of reference, moving with velocity $0 \cdot 8c$ in a direction (i) parallel to its length and (ii) at an angle of 30° with its length. What is the orientation of the rod in the moving frame of reference in case (ii)? 20

2016

Q9:

Given a proton for which $\beta = 0 \cdot 995$ measured in the laboratory. What are the corresponding relativistic energy and momentum? Take, $m_p = 1 \cdot 67 \times 10^{-24}$ g. 10

2016

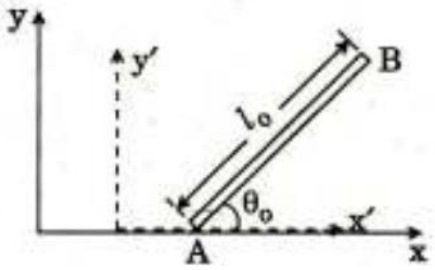
Q10:

Prove that $x^2 + y^2 + z^2 = c^2 t^2$ is invariant under Lorentz transformation. 10

2017

Q11:

A rod of length l_0 is kept at rest in x′y′ plane of its rest frame making an angle θ_0 with x′ axis. What is the length and orientation of the rod in a laboratory frame (x, y) in which the rod moves to the right with velocity v ? *15*



2018

Q12:

A reference frame S′ moves with respect to rest frame S with a uniform velocity 'v' parallel to x-direction. Show from Lorentz transformation that two events simultaneous ($t_1 = t_2$) at different positions ($x_1 \neq x_2$) in S frame are not in general simultaneous in S′ frame.

2019

Q13:

The mean life of π meson is 2×10^{-8} s. Calculate the mean life of a meson moving with a velocity of 0·8 c, where c is the velocity of light. *15*

2019

Q14:

Two β-particles A and B emitted by a radioactive source R travel in opposite directions, each with a velocity of 0·9 c with respect to the source. Find the velocity of B with respect to A (Here c is the velocity of light). *15*

2019

Q15:

What do you understand by length contraction ? Calculate the percentage length contraction of a rod moving with a velocity 0·8 c in a direction inclined at 60° with respect to its own length. *15*

2020

Q16:

Derive the relativistic expression for kinetic energy by considering mass variation with velocity. Hence, establish the relation between momentum (p) and energy (E) for a relativistic particle; $\frac{dE}{dp} = v$. *20*

2020

Q17:

An observer detects two explosions, one that occurs near him at a certain time and another that occurs 2·0 ms later 100 km away. Another observer finds that the two explosions occur at the same place. What time interval separates the explosions to the second observer ? *10*

2021

Q18:

(i) Calculate the mass and momentum of a proton of rest mass $1{\cdot}67 \times 10^{-27}$ kg moving with a velocity of 0·8c, where c is the velocity of light. If it collides and sticks to a stationary nucleus of mass $5{\cdot}0 \times 10^{-26}$ kg, find the velocity of the resultant particle.

(ii) Calculate the mass of the particle whose kinetic energy is half of its total energy. Find the velocity with which the particle is travelling. *8+7*

2021

Q19:

Show that for very small velocity, the equation for kinetic energy, $K = \Delta m c^2$ becomes $K = \frac{1}{2} m_0 v^2$, where notations have their usual meanings. 10

2022

Q20:

Two spaceships approach each other, both moving with same speed as measured by a stationary observer on the Earth. Their relative speed is $0{\cdot}7c$. Determine the velocity of each spaceship as measured by the stationary observer on the Earth. 15

2022

Q21:

What are the consequences of Lorentz transformations on length and time when observed from a frame moving at relativistic velocities ? *10*

2023

Q22.

A charged π-meson with rest mass of $273\,m_e$ at rest decays into a neutrino and a μ-meson of rest mass $207\,m_e$. Find the kinetic energy of the μ-meson and the energy of the neutrino. (m_e is the rest mass of the electron) 10

2024

Q23.

Consider three inertial frames of reference O, O' and O''. Let O' move with a velocity V with respect to O and O'' move with a velocity V' with respect to O'. Both velocities are in the same direction. Write down the transformation equations relating x, y, z, t with x', y', z', t' and also those relating x', y', z', t' with x'', y'', z'', t''. Hence obtain the relations between x, y, z, t and x'', y'', z'', t''. (The direction of velocity is chosen along the x-axis as per convention) 15

2024

Q24.

A galaxy in the constellation Ursa Major is receding from the Earth at 15000 km/s. If one of the characteristic wavelengths of light emitted by the galaxy is 550 nm, what is the corresponding wavelength measured by astronomers on the Earth? 5

2024

Q25.

A particle of rest mass 1 kg and velocity of magnitude 0·9c collides with a particle of mass 2 kg at rest. After collision the two particles coalesce and form a single particle of mass M and velocity V. Determine M and V. *10*

2025

Q26.

Show that the electromagnetic wave equation is invariant under Lorentz transformations. *15*

2025