Tutorial 3: Electromagnetic Waves:

- Displacement current and Maxwell's equations; Wave equations in vacuum, Poynting theorem; Vector and scalar potentials;
- Electromagnetic field tensor, covariance of Maxwell's equations;
- Wave equations in isotropic dielectrics, reflection and refraction at the boundary of two dielectrics;
- Fresnel's relations; Total internal reflection;
- Normal and anomalous dispersion; Rayleigh scattering;

Q1:

The electric field of a plane e.m. wave travelling along the z-axis is

$$\vec{E} = (E_{ox}\hat{x} + E_{oy}\hat{y})\sin(\omega t - kz + \phi).$$

Determine the magnetic field. *10*

2013

Q2:

(a) (i) Considering an isotropic, linear, non-conducting, non-magnetic and inhomogeneous dielectric medium with

$$\vec{D} = \varepsilon\vec{E} = \varepsilon_0 n^2(x, y, z)\vec{E},$$

show that the electromagnetic wave equation for the field $\vec{E}$ is given by

$$\nabla^2\vec{E} + \vec{\nabla}\left(\frac{1}{n^2}\vec{\nabla}n^2.\vec{E}\right) - \mu_0\varepsilon_0 n^2\frac{\partial^2\vec{E}}{\partial t^2} = 0. \quad 15$$

(ii) Write down the scalar equation for E_x from the above equation. *5*

(iii) Interpret physically the situation if we move from homogeneous to an inhomogeneous medium. *5*

(iv) Obtain the similar vector equation for the magnetic field $\vec{H}$ in inhomogeneous medium. *15*

(b) For a uniform wire of length L and radius a having a potential difference V between the ends and a current I along it, calculate the energy per unit time delivered to the wire by Poynting vector. *10*

2013

Q3:

Starting from Maxwell's equation, obtain the wave equation for the electric field $\vec{E}$ in free space and appropriate wave equation for the electric field $\vec{E} = E_z(x, y, z)\,\hat{z}$. *10*

2014

Q4:

Show that the energy flow due to a plane electromagnetic wave propagating along z-direction in a dielectric medium is given by

$$\hat{z}\,\frac{k}{\omega\mu}\,E_o^2\cos^2(kz - \omega t),$$

where k and ω are the propagation vector and angular frequency, E_0 is electric field amplitude, μ is the relative permeability of the medium. *10*

2014

Q5:

Derive the equation that represents Poynting's theorem. What is its physical significance ? *20*

2015

Q6:

A radio station transmits electromagnetic waves isotropically with an average power of 200 kW. Determine the average magnitude of the maximum electric field at a distance of 5 km from it. *15*

2015

Q7:

A plane electromagnetic wave propagating along $+\hat{z}$ direction is incident normally on the boundary at z = 0 between medium A (z < 0) and medium B (z > 0). Determine the reflection coefficient and transmission coefficient for the wave. *20*

2015

Q8:

In free space, the electric field of electromagnetic wave is given as $\vec{E}(x, t) = 120\cos(\omega t - kx)\hat{y}$ V/m. Find the average power crossing a circular area of radius one metre in the *yz*-plane. 10

2016

Q9:

Write down Maxwell's equations for linear dielectrics and deduce the equation of continuity. 10

2016

Q10:

State and prove Poynting's theorem. 20

2016

Q11:

Write down the electromagnetic wave equations in non-conducting dielectric medium. Hence show that the velocity of wave propagation is given by $v = \sqrt{\frac{1}{\varepsilon\mu}}$, where the symbols have their usual meanings. 10

2017

Q12:

Write down the physical significance of Maxwell's equations and explain the concept of displacement current by using a proper example. 10

2017

Q13:

Define a plane electromagnetic wave. A plane polarized wave is incident on the interface between two dielectric media. Obtain expressions for the amplitudes of the reflected and transmitted waves when the incident wave is polarized with its electric field E vector perpendicular to the plane of incidence. Discuss the phase relationships of the reflected and transmitted waves with respect to the incident wave. *20*

2018

Q14:

Write down Maxwell's equations in integral form. Explain the significance of each of these equations. *5*

2018

Q15:

A parallel plate capacitor has plate area = 4·0 cm^2 and plate separation = 2·0 mm. An a.c. voltage V = 20 sin (5×10^3 t) volts is applied across the plates. If the dielectric constant of the medium between the plates is $\varepsilon_r = 2{\cdot}0$, calculate the displacement current. *5*

2018

Q16:

Find the values of E and H on the surface of a wire carrying a current. By computing the Poynting vector, show that it represents a flow of energy into the wire. *20*

2019

Q17:

For the electric field given by $E = E_0 e^{i\omega t}$, show that the conduction current is in phase with the electric field, while the displacement current leads the electric field by $\frac{\pi}{2}$ radians. Also, show that the displacement current in a good conductor is negligible compared to the conduction current at any frequency lower than the optical frequencies ($f < 10^{15}$ Hz). *10*

2020

Q18:

For free space show that electromagnetic (EM) wave is transverse in nature. Show that for free space, the total outward flux of EM energy through surface S bounding volume V is equal to the rate of loss of EM energy from the volume V.

A laser beam of 2 mm diameter has average power of 20 GW. Calculate the peak values of electric and magnetic fields in the laser beam. *20*

2020

Q19:

Write Maxwell's equations in free space in both differential and integral forms. Obtain wave equations and show that electromagnetic waves can travel in free space with a speed of light. Can one get the wave equations from the integral form of the Maxwell's equations ? *15*

2020

Q20:

In an inertial reference frame S there is only a uniform electric field $\vec{E} = 8\ \text{kVm}^{-1}$. Find the magnitude of $\vec{E}'$ and $\vec{B}'$ in the inertial reference frame S′ moving with a constant velocity $\vec{v}$ relative to the frame S at an angle $\alpha = 45°$ to the vector $\vec{E}$. The velocity of the frame S′ is 0·6 times the velocity of light c. *20*

2021

Q21:

Calculate the skin depth of electromagnetic waves of 1 MHz incident on a good conductor having $\sigma = 5{\cdot}8 \times 10^7\ \text{Sm}^{-1}$. Assume that inside the conductor $\mu = \mu_0 = 4\pi \times 10^{-7}\ \text{Hm}^{-1}$. *10*

2021

Q22:

Write down Maxwell's equations in a non-conducting medium with constant permeability and susceptibility ($\rho = j = 0$). Show that $\vec{E}$ and $\vec{B}$ each satisfies the wave equation, and find an expression for the wave velocity. Write down the plane wave solutions for $\vec{E}$ and $\vec{B}$, and show how $\vec{E}$ and $\vec{B}$ are related. 15

2022

Q23:

In a partially conducting medium, $\varepsilon_r = 18{\cdot}5$, $\mu_r = 800$ and $\sigma = 1\ \text{S m}^{-1}$. Find α, β, η and the velocity u, for a frequency of 10^9 Hz. Determine $\vec{H}(z, t)$. Given, $\vec{E}(z, t) = 50\, e^{-\alpha z} \cos(\omega t - \beta a_z) a_y\ \text{V m}^{-1}$. 20

2022

Q24:

Show that Continuity equation is embedded in Maxwell's equations. *10*

2023

Q25:

Consider the interaction of an electromagnetic wave at the interface of two dielectric media. If electric field $\vec{E}$ is parallel to the plane of incidence, obtain Fresnel's equations and Brewster's law of polarization. *20*

2023

Q26.

Using Maxwell's equations, obtain Poisson's equation and Laplace's equation. The region $-\frac{\pi}{2} < \frac{z}{z_0} < \frac{\pi}{2}$ has a charge density $\rho = 10^{-8} \cos\left(\frac{z}{z_0}\right) (C/m^3)$. Elsewhere the charge density is zero. Find the electric potential V and electric field E from the Poisson's equation. 15

2024

Q27.

A uniform plane wave with $\vec{E} = E_x \hat{a}_x$ propagates in a lossless medium ($\varepsilon_r = 4$, $\mu_r = 1$, $\sigma = 0$) in the z-direction. Assume that E_x is sinusoidal with a frequency 100 MHz and has a maximum value of 10^{-4} (V/m) at $t = 0$ and $z = \frac{1}{8}$ (m).

(i) Write the expression for instantaneous E for any t and z.

(ii) Write the expression for instantaneous H.

(iii) Determine the locations where E_x is a positive maximum, when $t = 10^{-8}$ (s). 20

2024

Q28.

What is anomalous dispersion? How does the phenomenon of dispersion lead to the separation of white light into its constituent colours? 5

2024

Q29.

(a) (i) मुक्त आकाश में एक विद्युत-क्षेत्र ($\vec{E}$) निम्नलिखित व्यंजक द्वारा व्यक्त किया गया है :

$$\vec{E} = 10 \cos(\omega t - 100 x)\hat{j} \text{ V/m}$$

कोणीय आवृत्ति ω और विस्थापन धारा ज्ञात कीजिए।

In free space, an electric field ($\vec{E}$) is given by the following expression :

$$\vec{E} = 10 \cos(\omega t - 100 x)\hat{j} \text{ V/m}$$

Find the angular frequency ω and the displacement current. *10*

(ii) एक विद्युत-चुम्बकीय तरंग का चुम्बकीय क्षेत्र $|\vec{B}| = 55 \times 10^{-8}$ T है। प्वाइंटिंग सदिश का परिमाण ज्ञात कीजिए।

An electromagnetic wave has its magnetic field $|\vec{B}| = 55 \times 10^{-8}$ T. Determine the magnitude of the Poynting vector. *5*

2025